59. τ-Lepton Decay Parameters

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The purpose of the measurements of the decay parameters (also known as Michel parameters) of the τ is to determine the structure (spin and chirality) of the current mediating its decays.

59.1 Leptonic Decays:

The Michel parameters are extracted from the energy spectrum of the charged daughter lepton $\ell = e, \mu$ in the decays $\tau \to \ell \nu_\ell \nu_\tau$. Ignoring radiative corrections, neglecting terms of order $(m_\ell/m_\tau)^2$ and $(m_\tau/\sqrt{s})^2$, and setting the neutrino masses to zero, the spectrum in the laboratory frame reads

$$\frac{d\Gamma}{dx} = \frac{G_{\tau\ell}^2\, m_\tau^5}{192\, \pi^3} \left\{ f_0\,(x) + \rho f_1\,(x) + \eta \frac{m_\ell}{m_\tau} f_2\,(x) - P_\tau \left[\xi g_1\,(x) + \xi\delta g_2\,(x) \right] \right\} \tag{59.1}$$

with

$$\begin{aligned} & f_0(x) = 2 - 6\,x^2 + 4\,x^3 & & \\ & f_1(x) = -\tfrac{4}{9} + 4x^2 - \tfrac{32}{9}x^3 & & g_1(x) = -\tfrac{2}{3} + 4x - 6x^2 + \tfrac{8}{3}x^3 \\ & f_2(x) = 12\,(1-x)^2 & & g_2(x) = \tfrac{4}{9} - \tfrac{16}{3}x + 12x^2 - \tfrac{64}{9}\,x^3\,. \end{aligned} \tag{59.2}$$

The quantity x is the fractional energy of the daughter lepton ℓ, *i.e.*, $x = E_\ell/E_{\ell,max} \approx E_\ell/(\sqrt{s}/2)$ and P_τ is the polarization of the tau leptons. The integrated decay width is given by

$$\Gamma = \frac{G_{\tau\ell}^2\, m_\tau^5}{192\, \pi^3} \left(1 + 4\,\eta\, \frac{m_\ell}{m_\tau} \right) \,. \tag{59.3}$$

The situation is similar to muon decays $\mu \to e \nu_e \nu_\mu$. The generalized matrix element with the couplings $g^\gamma_{\varepsilon\mu}$ and their relations to the Michel parameters ρ, η, ξ, and δ have been described in the "Note on Muon Decay Parameters". The Standard Model expectations are 3/4, 0, 1, and 3/4, respectively. For more details, see [1].

More information can be extracted from the decays, if the polarization of the daughter lepton is measured. Belle presented a first result (see [2]). A new parameter ξ' is introduced. Its relation to the couplings is given by (see [3]):

$$\xi' = 1 - 2 \sum_{\omega = L,R} \left(\frac{1}{4} \left| g^S_{R\omega} \right|^2 + \left| g^V_{R\omega} \right|^2 \right) + 3 \left| g^T_{RL} \right|^2 \,. \tag{59.4}$$

The new measurement improves the limit on g^T_{RL} by about a factor of two.

Additional Michel parameters can be defined in radiative decays $\tau \to \ell \nu_\ell \nu_\tau \gamma$, see [4]. A first measurement is presented in [5]. The experimental precision does not provide a significant impact on the knowledge about the couplings, yet.

59.2 Hadronic Decays:

In the case of hadronic decays $\tau \to h \nu_\tau$, with $h = \pi$, ρ, or a_1, the ansatz is restricted to purely vectorial currents. The matrix element is

$$\frac{G_{\tau h}}{\sqrt{2}} \sum_{\lambda = R,L} g_\lambda \,\langle\, \overline{\Psi}_\omega(\nu_\tau) \mid \gamma^\mu \mid \Psi_\lambda(\tau)\, \rangle \;\; J^h_\mu \tag{59.5}$$

with the hadronic current J^h_μ. The neutrino chirality ω is uniquely determined from λ. The spectrum depends only on a single parameter ξ_h

$$\frac{d^n\Gamma}{dx_1 dx_2 \ldots dx_n} = f\left(\vec{x}\right) + \xi_h P_\tau g\left(\vec{x}\right)\ , \tag{59.6}$$

with f and g being channel-dependent functions of the n observables $\vec{x} = (x_1, x_2, \ldots, x_n)$ (see [6]). The parameter ξ_h is related to the couplings through

$$\xi_h = |g_L|^2 - |g_R|^2\ . \tag{59.7}$$

ξ_h is the negative of the chirality of the τ neutrino in these decays. In the Standard Model, $\xi_h = 1$. Also included in the Data Listings for ξ_h are measurements of the neutrino helicity which coincide with ξ_h, if the neutrino is massless (ASNER 00 [7], ACKERSTAFF 97R [8], AKERS 95P [9], ALBRECHT 93C [10], and ALBRECHT 90I [11]).

59.3 Combination of Measurements:

The individual measurements are combined, taking into account the correlations between the parameters. In a first fit, universality between the two leptonic decays, and between all hadronic decays, is assumed. A second fit is made without these assumptions. The results of the two fits are provided as OUR FIT in the Data Listings below in the tables whose title includes "(e or mu)" or "(all hadronic modes)", and "(e)", "(mu)" *etc.*, respectively. The measurements show good agreement with the Standard Model. The χ^2 values with respect to the Standard model predictions are 24.1 for 41 degrees of freedom and 26.8 for 56 degrees of freedom, respectively. The correlations are reduced through this combination to less than 20%, with the exception of ρ and η which are correlated by +23%, for the fit with universality and by +70% for $\tau \to \mu\nu_\mu\nu_\tau$.

59.4 Model-independent Analysis:

From the Michel parameters, limits can be derived on the couplings $g^\kappa_{\varepsilon\lambda}$ without further model assumptions. In the Standard model $g^V_{LL} = 1$ (leptonic decays), and $g_L = 1$ (hadronic decays) and all other couplings vanish. First, the partial decay widths have to be compared to the Standard Model predictions to derive limits on the normalization of the couplings $A_x = G^2_{\tau x}/G^2_F$ with Fermi's constant G_F:

$$\begin{aligned} A_e &= 1.0022 \pm 0.0041\ , \\ A_\mu &= 0.983 \pm 0.017\ , \\ A_\pi &= 0.9957 \pm 0.0049\ . \end{aligned} \tag{59.8}$$

Then limits on the couplings (95% CL) can be extracted (see [12] and [13]). Without the assumption of universality, the limits given in Table 59.1 are derived.

59.5 Model-dependent Interpretation:

More stringent limits can be derived assuming specific models. For example, in the framework of a two Higgs doublet model, the measurements correspond to a limit of $m_{H^\pm} > 2.0\ \text{GeV} \times \tan\beta$ on the mass of the charged Higgs boson, or a limit of 500 GeV on the mass of the second W boson in left-right symmetric models for arbitrary mixing (both 95% CL). See [13] and [14].

Table 59.1: Coupling constants $g^{\gamma}_{\varepsilon\mu}$. 95% confidence level experimental limits. The limits include the quoted values of A_e, A_μ, and A_π and assume $A_\rho = A_{a_1} = 1$.

$\tau \to e\nu_e\nu_\tau$		
$\lvert g^S_{RR}\rvert < 0.70$	$\lvert g^V_{RR}\rvert < 0.17$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.99$	$\lvert g^V_{LR}\rvert < 0.13$	$\lvert g^T_{LR}\rvert < 0.082$
$\lvert g^S_{RL}\rvert < 2.01$	$\lvert g^V_{RL}\rvert < 0.52$	$\lvert g^T_{RL}\rvert < 0.51$
$\lvert g^S_{LL}\rvert < 2.01$	$\lvert g^V_{LL}\rvert < 1.004$	$\lvert g^T_{LL}\rvert \equiv 0$
$\tau \to \mu\nu_\mu\nu_\tau$		
$\lvert g^S_{RR}\rvert < 0.72$	$\lvert g^V_{RR}\rvert < 0.18$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.95$	$\lvert g^V_{LR}\rvert < 0.12$	$\lvert g^T_{LR}\rvert < 0.079$
$\lvert g^S_{RL}\rvert < 2.01$	$\lvert g^V_{RL}\rvert < 0.52$	$\lvert g^T_{RL}\rvert < 0.27$
$\lvert g^S_{LL}\rvert < 2.01$	$\lvert g^V_{LL}\rvert < 1.006$	$\lvert g^T_{LL}\rvert \equiv 0$
$\tau \to \pi\nu_\tau$		
$\lvert g^V_R\rvert < 0.15$	$\lvert g^V_L\rvert > 0.984$	
$\tau \to \rho\nu_\tau$		
$\lvert g^V_R\rvert < 0.10$	$\lvert g^V_L\rvert > 0.995$	
$\tau \to a_1\nu_\tau$		
$\lvert g^V_R\rvert < 0.16$	$\lvert g^V_L\rvert > 0.987$	

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