75. B^0–$\overline{B}^0$ Mixing

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There are two neutral B^0–$\overline{B}^0$ meson systems, B^0_d–$\overline{B}^0_d$ and B^0_s–$\overline{B}^0_s$ (generically denoted B^0_q–$\overline{B}^0_q$, $q = s, d$), which exhibit particle-antiparticle mixing [1]. This mixing phenomenon is described in Ref. [2]. In the following, we adopt the notation introduced in Ref. [2], and assume CPT conservation throughout. In each system, the light (L) and heavy (H) mass eigenstates,

$$|B_{\rm L,H}\rangle = p|B^0_q\rangle \pm q|\overline{B}^0_q\rangle, \tag{75.1}$$

have a mass difference $\Delta m_q = m_{\rm H} - m_{\rm L} > 0$, a total decay width difference $\Delta\Gamma_q = \Gamma_{\rm L} - \Gamma_{\rm H}$ and an average decay width $\Gamma_q = (\Gamma_{\rm L} + \Gamma_{\rm H})/2$. In the absence of CP violation in the mixing, $|q/p| = 1$, the differences are given by $\Delta m_q = 2|M_{12}|$ and $|\Delta\Gamma_q| = 2|\Gamma_{12}|$, where M_{12} and Γ_{12} are the off-diagonal elements of the mass and decay matrices [2]. The evolution of a pure $|B^0_q\rangle$ or $|\overline{B}^0_q\rangle$ state at $t = 0$ is given by

$$|B^0_q(t)\rangle = g_+(t)\,|B^0_q\rangle + \frac{q}{p}\,g_-(t)\,|\overline{B}^0_q\rangle, \tag{75.2}$$

$$|\overline{B}^0_q(t)\rangle = g_+(t)\,|\overline{B}^0_q\rangle + \frac{p}{q}g_-(t)\,|B^0_q\rangle, \tag{75.3}$$

which means that the flavor states remain unchanged (+) or oscillate into each other (−) with time-dependent probabilities proportional to

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma_q t}}{2}\left[\cosh\left(\frac{\Delta\Gamma_q}{2}\,t\right) \pm \cos(\Delta m_q\,t)\right]. \tag{75.4}$$

In the absence of CP violation, the time-integrated mixing probability $\int|g_-(t)|^2\,dt/(\int|g_-(t)|^2\,dt + \int|g_+(t)|^2\,dt)$ is given by

$$\chi_q = \frac{x_q^2 + y_q^2}{2(x_q^2+1)}\,, \quad \text{where} \quad x_q = \frac{\Delta m_q}{\Gamma_q}\,, \quad y_q = \frac{\Delta\Gamma_q}{2\Gamma_q}\,. \tag{75.5}$$

75.1 Standard Model predictions and phenomenology

In the Standard Model, the transitions $B^0_q \to \overline{B}^0_q$ and $\overline{B}^0_q \to B^0_q$ are due to the weak interaction. They are described, at the lowest order, by box diagrams involving two W bosons and two up-type quarks (see Fig. 75.1), as is the case for $K^0 - \overline{K}^0$ mixing. However, the long range interactions arising from intermediate virtual states are negligible for the neutral B meson systems, because the large B mass is off the region of hadronic resonances. The calculation of the dispersive and absorptive parts of the box diagrams yields the following predictions for the off-diagonal element of the mass and decay matrices [3],

$$M_{12} = -\frac{G_F^2 m_W^2 \eta_B m_{B_q} B_{B_q} f_{B_q}^2}{12\pi^2}\,S_0(m_t^2/m_W^2)\,(V_{tq}^* V_{tb})^2\,, \tag{75.6}$$

$$\Gamma_{12} = \frac{G_F^2 m_b^2 \eta'_B m_{B_q} B_{B_q} f_{B_q}^2}{8\pi} \times\left[(V_{tq}^*V_{tb})^2 + V_{tq}^*V_{tb}V_{cq}^*V_{cb}\ \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right) + (V_{cq}^*V_{cb})^2\ \mathcal{O}\left(\frac{m_c^4}{m_b^4}\right)\right], \tag{75.7}$$

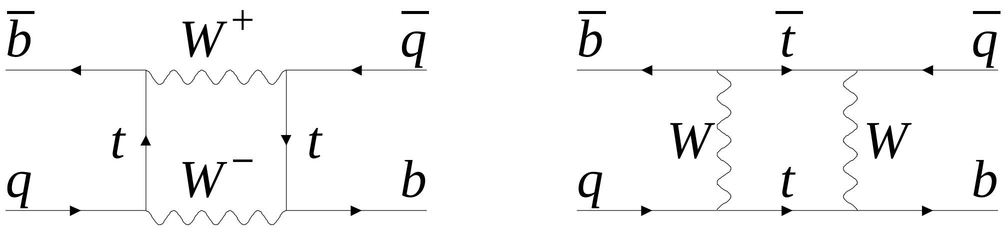

Figure 75.1: Dominant box diagrams for the $B_q^0 \to \overline{B_q}^0$ transitions ($q = d$ or s). Similar diagrams exist where one or both t quarks are replaced with c or u quarks.

where G_F is the Fermi constant, m_W the W boson mass, and m_i the mass of quark i; m_{B_q}, f_{B_q} and B_{B_q} are the B_q^0 mass, weak decay constant and bag parameter, respectively. The known function $S_0(x_t)$ can be approximated very well by $0.784\, x_t^{0.76}$ [4], and V_{ij} are the elements of the CKM matrix [5]. The QCD corrections η_B and η'_B are of order unity. The only non-negligible contributions to M_{12} are from box diagrams involving two top quarks. The phases of M_{12} and Γ_{12} satisfy

$$\phi_M - \phi_\Gamma = \pi + \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right) , \tag{75.8}$$

implying that the mass eigenstates have mass and width differences of opposite signs. This means that, like in the K^0–$\overline{K}^0$ system, the heavy state is expected to have a smaller decay width than that of the light state: $\Gamma_{\rm H} < \Gamma_{\rm L}$. Hence, $\Delta\Gamma_q = \Gamma_{\rm L} - \Gamma_{\rm H}$ is expected to be positive in the Standard Model.

Furthermore, the quantity

$$\left|\frac{\Gamma_{12}}{M_{12}}\right| \simeq \frac{3\pi}{2}\frac{m_b^2}{m_W^2}\frac{1}{S_0(m_t^2/m_W^2)} \sim \mathcal{O}\left(\frac{m_b^2}{m_t^2}\right) \tag{75.9}$$

is small, and a power expansion of $|q/p|^2$ yields

$$\left|\frac{q}{p}\right|^2 = 1 + \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin(\phi_M - \phi_\Gamma) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right) . \tag{75.10}$$

Therefore, considering both Eqs. (75.8) and (75.9), the CP-violating parameter

$$1 - \left|\frac{q}{p}\right|^2 \simeq \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \tag{75.11}$$

is expected to be very small: $\sim \mathcal{O}(10^{-3})$ for the B_d^0–$\overline{B}_d^0$ system and $\lesssim \mathcal{O}(10^{-4})$ for the B_s^0–$\overline{B}_s^0$ system [6].

In the approximation of negligible CP violation in mixing, the ratio $\Delta\Gamma_q/\Delta m_q$ is equal to the small quantity $|\Gamma_{12}/M_{12}|$ of Eq. (75.9); it is hence independent of CKM matrix elements, *i.e.*, the same for the B_d^0–$\overline{B}_d^0$ and B_s^0–$\overline{B}_s^0$ systems. Calculations [7] yield $\sim 5 \times 10^{-3}$ with a $\sim 15\%$ uncertainty. Given the published experimental knowledge [8] on the mixing parameter x_q

$$\begin{cases} x_d = 0.7697 \pm 0.0035 & (B_d^0\text{–}\overline{B}_d^0 \text{ system}) \\ x_s = 26.99 \pm 0.09 & (B_s^0\text{–}\overline{B}_s^0 \text{ system}) \end{cases} , \tag{75.12}$$

the Standard Model thus predicts that $\Delta\Gamma_d/\Gamma_d$ is very small (below 1%), but $\Delta\Gamma_s/\Gamma_s$ considerably larger ($\sim$ 10%). These width differences are caused by the existence of final states to which both the B_q^0 and $\overline{B}_q^0$ mesons can decay. Such decays involve $b \to c\bar{c}q$ quark-level transitions, which are Cabibbo-suppressed if $q = d$ and Cabibbo-allowed if $q = s$.

A complete set of Standard Model predictions for all mixing parameters in both the B_d^0–$\overline{B}_d^0$ and B_s^0–$\overline{B}_s^0$ systems can be found in Ref. [7].

75.2 Experimental issues and methods for oscillation analyses

Time-integrated measurements of B^0–$\overline{B}^0$ mixing were published for the first time in 1987 by UA1 [9] and ARGUS [10], and since then by many other experiments. These measurements are typically based on counting same-sign and opposite-sign lepton pairs from the semileptonic decay of the produced $b\bar{b}$ pairs. Such analyses cannot easily separate the contributions from the different b-hadron species, therefore, the clean environment of $\Upsilon(4S)$ machines (where only B_d^0 and charged B_u mesons are produced) is in principle best suited to measure χ_d.

However, better sensitivity is obtained from time-dependent analyses aiming at the direct measurement of the oscillation frequencies Δm_d and Δm_s, from the proper time distributions of B_d^0 or B_s^0 candidates identified through their decay in (mostly) flavor-specific modes, and suitably tagged as mixed or unmixed. This is particularly true for the B_s^0–$\overline{B}_s^0$ system, where the large value of x_s implies maximal mixing, *i.e.*, $\chi_s \simeq 1/2$. In such analyses, the B_d^0 or B_s^0 mesons are either fully reconstructed, partially reconstructed from a charm meson, selected from a lepton with the characteristics of a $b \to \ell^-$ decay, or selected from a reconstructed displaced vertex. At high-energy colliders (LEP, SLC, Tevatron, LHC), the proper time $t = \frac{m_B}{p}L$ is measured from the distance L between the production vertex and the B decay vertex, and from an estimate of the B momentum p. At asymmetric B factories (SuperKEKB, KEKB, PEP-II), producing $e^+e^- \to \Upsilon(4S) \to B_d^0\,\overline{B_d}^0$ events with a boost $\beta\gamma$ (= 0.28, 0.425, 0.55), the proper time difference between the two B candidates is estimated as $\Delta t \simeq \frac{\Delta z}{\beta\gamma c}$, where Δz is the spatial separation between the two B decay vertices along the boost direction. In all cases, the good resolution needed on the vertex positions is obtained with silicon detectors.

The average statistical significance $\mathcal{S}$ of a B_q^0 oscillation signal can be approximated as [11]

$$\mathcal{S} \approx \sqrt{N/2}\, f_{\rm sig}\,(1-2\eta)\, e^{-(\Delta m_q\,\sigma_t)^2/2}\,, \tag{75.13}$$

where N is the number of selected and tagged candidates, $f_{\rm sig}$ is the fraction of signal in that sample, η is the total mistag probability, and σ_t is the resolution on proper time (or proper time difference). The quantity $\mathcal{S}$ decreases very quickly as Δm_q increases; this dependence is controlled by σ_t, which is therefore a critical parameter for Δm_s analyses. At high-energy colliders, the proper time resolution $\sigma_t \sim \frac{m_B}{\langle p\rangle}\sigma_L \oplus t\frac{\sigma_p}{p}$ includes a constant contribution due to the decay length resolution σ_L (typically 0.04–0.3 ps), and a term due to the relative momentum resolution σ_p/p (typically 10–20% for partially reconstructed decays), which increases with proper time. At B factories, the boost of the B mesons is estimated from the known beam energies, and the term due to the spatial resolution dominates (typically 0.7–1.5 ps because of the much smaller B boost).

In order to tag a B_q^0 candidate as mixed or unmixed, it is necessary to determine its flavor both in the initial state and in the final state. The initial and final state mistag probabilities, η_i and η_f, degrade $\mathcal{S}$ by a total factor $(1-2\eta) = (1-2\eta_i)(1-2\eta_f)$. In lepton-based analyses, the final state is tagged by the charge of the lepton from $b \to \ell^-$ decays; the largest contribution to η_f is then due to $\bar{b} \to \bar{c} \to \ell^-$ decays. Alternatively, the charge of a reconstructed charm meson (D^{*-} from B_d^0 or D_s^- from B_s^0), or that of a kaon hypothesized to come from a $b \to c \to s$ decay [12], can be used. For fully-inclusive analyses based on topological vertexing, final-state tagging techniques include jet-charge [13] and charge-dipole [14,15] methods. At high-energy colliders, the methods to tag the initial state (*i.e.*, the state at production), can be divided into two groups: the ones that tag the initial charge of the $\bar{b}$ quark contained in the B_q^0 candidate itself (same-side tag), and the ones that tag the initial charge of the other b quark produced in the event (opposite-side tag). On the same side, the sign of a charged pion, kaon or proton from the primary vertex is correlated with the production state of the B_q^0 meson if that particle is a decay product of a B^{**} state or the first in the fragmentation chain [16,17]. Jet- and vertex-charge techniques work on both sides and on the opposite side, respectively. Finally, the charge of a lepton from $b \to \ell^-$, of a kaon from $b \to c \to s$ or of a charm hadron from $b \to c$ [18] can be used as an opposite-side tag, keeping in mind that its performance is degraded due to integrated mixing. At SLC, the beam polarization produced a sizeable forward-backward asymmetry in the $Z \to b\bar{b}$ decays, and provided another very interesting and effective initial state tag based on the polar angle of the B_q^0 candidate [14]. Initial state tags have also been combined to reach $\eta_i \sim 26\%$ at LEP [17,19] or 22% at SLD [14] with full efficiency. In the case $\eta_f = 0$, this corresponds to an effective tagging efficiency $Q = \epsilon D^2 = \epsilon(1-2\eta)^2$, where ϵ is the tagging efficiency, in the range $23-31\%$. The equivalent figure achieved by CDF during Tevatron Run I was $\sim 3.5\%$ (see tagging summary on page 160 of Ref. [20]), reflecting the fact that tagging is more difficult at hadron colliders. The CDF and DØ analyses of Tevatron Run II data reached $\epsilon D^2 = (1.8 \pm 0.1)\%$ [21] and $(2.5 \pm 0.2)\%$ [22] for opposite-side tagging, while same-side kaon tagging (for B_s^0 analyses) contributed an additional $3.7-4.8\%$ at CDF [21], and pushed the combined performance to $(4.7 \pm 0.5)\%$ at DØ [23]. LHCb, operating in the forward region at the LHC where the environment is different in terms of track multiplicity and b-hadron production kinematics, has reported $\epsilon D^2 = (2.10 \pm 0.25)\%$ [24] for opposite-side tagging, $(1.80 \pm 0.26)\%$ [25] for same-side kaon tagging, and $(2.11 \pm 0.11)\%$ [26] for same-side pion and proton tagging: the combined figure ranges typically between $(3.73 \pm 0.15)\%$ [27] and $(6.3 \pm 0.5)\%$ [28] depending on the mode in which the tagged B_s^0 meson is reconstructed, and reaches up to $(8.1 \pm 0.6)\%$ [29] for hadronic B_d^0 modes. ATLAS [30] and CMS [31] have reported $\epsilon D^2 \sim 1.75\%$ and $\epsilon D^2 \sim 10\%$ using opposite-side tagging of $B_s^0 \to J/\psi\phi$ decays.

At B factories, the flavor of a B_d^0 meson at production cannot be determined, since the two neutral B mesons produced in a $\Upsilon(4S)$ decay evolve in a coherent P-wave state where they keep opposite flavors at any time. However, as soon as one of them decays, the other follows a time-evolution given by Eqs. (75.2) or (75.3), where t is replaced with Δt (which will take negative values half of the time). Hence, the "initial state" tag of a B can be taken as the final-state tag of the other B. Effective tagging efficiencies of 30% are achieved by BaBar and Belle [32], using different techniques including $b \to \ell^-$ and $b \to c \to s$ tags. It is worth noting that, in this case, mixing of the other B (*i.e.*, the coherent mixing occurring before the first B decay) does not contribute to the mistag probability.

Before the experimental observation of a decay-width difference, oscillation analyses typically neglected $\Delta\Gamma_q$ in Eq. (75.4), and described the time dependence with the functions $\Gamma_q e^{-\Gamma_q t}(1 \pm \cos(\Delta m_q t))/2$ (high-energy colliders) or $\Gamma_d e^{-\Gamma_d |\Delta t|}(1 \pm \cos(\Delta m_d \Delta t))/4$ (asymmetric $\Upsilon(4S)$ machines). As can be seen from Eq. (75.4), a non-zero value of $\Delta\Gamma_q$ would effectively reduce the oscillation amplitude with a small time-dependent factor that would be very difficult to distinguish from time resolution effects. Measurements of Δm_q are usually extracted from the data using a maximum likelihood fit.

75.3 Δm_d and $\Delta\Gamma_d$ measurements

Many B_d^0–$\overline{B}_d^0$ oscillations analyses have been published [33] by the ALEPH [34], DELPHI [15, 35], L3 [36], OPAL [37,38], CDF [16], DØ [22], BaBar [39], Belle [40], Belle II [41], and LHCb [42–45] collaborations. Although a variety of different techniques have been used, the individual

Δm_d results obtained at LEP and Tevatron have remarkably similar precision. Their average is compatible with the more precise measurements at the asymmetric B factories and the LHC. The systematic uncertainties are not negligible; they are often dominated by sample composition, mistag probability, or b-hadron lifetime contributions. Before being combined, the measurements are adjusted on the basis of a common set of input values, including the b-hadron lifetimes and fractions published in this *Review*. Some measurements are statistically correlated. Systematic correlations arise both from common physics sources (fragmentation fractions, lifetimes, branching ratios of b hadrons), and from purely experimental or algorithmic effects (efficiency, resolution, tagging, background description). Combining all measurements [15,16,22,34–45] and accounting for all identified correlations yields $\Delta m_d = 0.5069 \pm 0.0016(\text{stat}) \pm 0.0011(\text{syst})\ \text{ps}^{-1}$ [8], a result dominated by the latest LHCb measurement with $B^0 \to D^{(*)-}\mu^+\nu_\mu X$ decays [45].

On the other hand, ARGUS and CLEO have published time-integrated measurements [46–48], which average to $\chi_d = 0.182 \pm 0.015$. Following Ref. [48], the width difference $\Delta\Gamma_d$ could in principle be extracted from the measured value of $1/\Gamma_d$ and the above averages for Δm_d and χ_d (see Eq. (75.5)), provided that $\Delta\Gamma_d$ has a negligible impact on the Δm_d and $1/\Gamma_d$ analyses that have assumed $\Delta\Gamma_d = 0$. However, $\Delta\Gamma_d/\Gamma_d$ is too small and the knowledge of χ_d too imprecise to provide useful sensitivity on $\Delta\Gamma_d/\Gamma_d$. Direct time-dependent studies published by DELPHI [15], BaBar [49], Belle [50], LHCb [51], ATLAS [52] and CMS [53] provide stronger constraints, which can be combined to yield [8]

$$\Delta\Gamma_d/\Gamma_d = +0.001 \pm 0.010 \,. \tag{75.14}$$

This determination is compatible both with zero and with the Standard Model prediction of $(4.0 \pm 0.9) \times 10^{-3}$ [54].

Assuming $\Delta\Gamma_d = 0$ and no CP violation in mixing, and using the B_d^0 lifetime average of 1.517 ± 0.004 ps [8], the Δm_d and χ_d results are combined to yield the world average

$$\Delta m_d = 0.5069 \pm 0.0019\ \text{ps}^{-1} \tag{75.15}$$

or, equivalently,

$$\chi_d = 0.1860 \pm 0.0011 \,. \tag{75.16}$$

This Δm_d value provides an estimate of $2|M_{12}|$, and can be used with Eq. (75.6) to extract $|V_{td}|$ within the Standard Model [55]. The main experimental uncertainties on the result come from m_t and Δm_d, but are still completely negligible with respect to the uncertainty due to the hadronic matrix element $f_{B_d}\sqrt{B_{B_d}} = 225 \pm 9$ MeV [56] obtained from three-flavor lattice QCD calculations.

75.4 Δm_s and $\Delta\Gamma_s$ measurements

After many years of intense search at LEP and SLC, B_s^0–$\overline{B}_s^0$ oscillations were first observed in 2006 by CDF using 1 fb^{-1} of Tevatron Run II data [21]. LHCb then observed B_s^0–$\overline{B}_s^0$ oscillations independently with $B_s^0 \to D_s^-\pi^+$ [42, 57], $B_s^0 \to D_s^-\mu^+\nu X$ [44] and $B_s^0 \to J/\psi K^+K^-$ [27] decays using up to 3 fb^{-1} of LHC Run 1 data. More recently measurements based on additional LHC Run 2 data have been published by CMS with $B_s^0 \to J/\psi\phi$ decays [31], and by LHCb with $B_s^0 \to D_s^-\pi^+\pi^-\pi^+$ [58], $B_s^0 \to D_s^-\pi^+$ [59] and $B_s^0 \to J/\psi K^+K^-$ [60] decays. Taking systematic correlations into account, the average [8] of all published measurements of Δm_s [21,27,31,42,44,57–60] is

$$\Delta m_s = 17.765 \pm 0.004(\text{stat}) \pm 0.004(\text{syst})\ \text{ps}^{-1} \,, \tag{75.17}$$

with an impressive precision dominated by the most recent $B_s^0 \to D_s^-\pi^+$ result [59] (see Fig. 75.2).

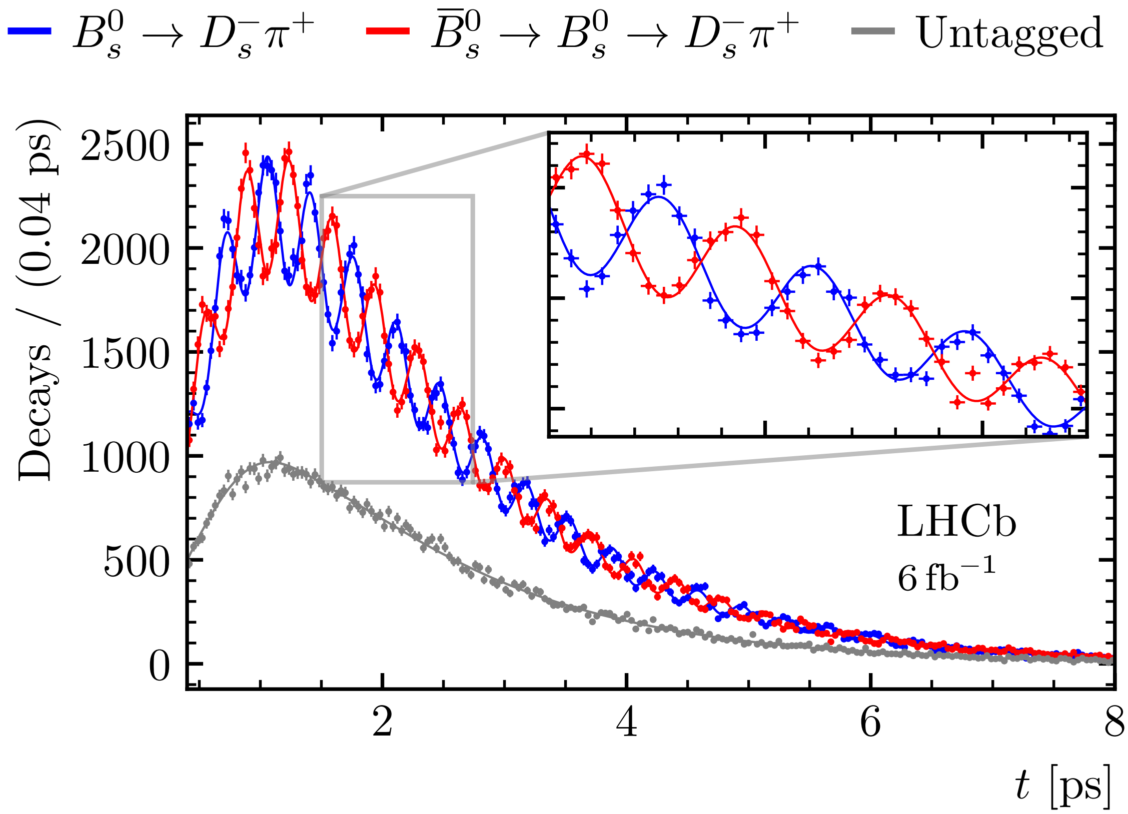

Figure 75.2: Proper decay-time distributions of background-subtracted $B^0_s \to D^-_s\pi^+$ decays tagged as unmixed (blue), tagged as mixed (red) or untagged (grey) in the Run 2 data of the LHCb experiment, displaying B^0_s–$\overline{B}^0_s$ oscillations [59].

The information on $|V_{ts}|$ obtained in the framework of the Standard Model is hampered by the hadronic uncertainty, as in the B^0_d case. However, several uncertainties cancel in the frequency ratio

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}}\,\xi^2\left|\frac{V_{ts}}{V_{td}}\right|^2 , \tag{75.18}$$

where the SU(3) flavor-symmetry breaking factor $\xi = (f_{B_s}\sqrt{B_{B_s}})/(f_{B_d}\sqrt{B_{B_d}})$ is obtained as 1.206 ± 0.017 from a combination of three-flavor lattice QCD calculations [56] dominated by the results of Ref. [61], or as $1.2014^{+0.0065}_{-0.0072}$ from QCD sum rules [62]. Using the measurements of Eqs. (75.15) and (75.17), one can extract

$$\left|\frac{V_{td}}{V_{ts}}\right| = \begin{cases} 0.2054 \pm 0.0004 \pm 0.0029\,(\text{lattice QCD}) \\ 0.2045 \pm 0.0004^{+0.0011}_{-0.0012}\,(\text{QCD sum rules}) \end{cases} , \tag{75.19}$$

in good agreement with (but much more precise than) the value obtained from the ratio of the $b \to d\gamma$ and $b \to s\gamma$ transition rates observed at the B factories [55].

The CKM matrix can be constrained using experimental results on observables such as Δm_d, Δm_s, $|V_{ub}/V_{cb}|$, ϵ_K, and $\sin(2\beta)$ together with theoretical inputs and unitarity conditions [55,63,64]. The constraint from our knowledge on the ratio $\Delta m_s/\Delta m_d$ is more effective in limiting the position of the apex of the CKM unitarity triangle than the one obtained from the Δm_d measurements alone, due to the reduced hadronic uncertainty in Eq. (75.18). We also note that the experimental value of Δm_s from Eq. (75.17) is consistent with the Standard Model predictions obtained from CKM fits where no experimental information on Δm_s is used, *e.g.*, 17.89 ± 0.65 ps^{-1} [63] or $17.26\,^{+0.63}_{-0.41}$ ps^{-1} [64]. It is also consistent with the prediction 18.23 ± 0.62 ps^{-1} from Ref. [7].

Information on $\Delta\Gamma_s$ can be obtained from the study of the proper time distribution of untagged B^0_s samples [65]. In the case of an inclusive B^0_s selection [66], or a flavor-specific (semileptonic or hadronic) B^0_s decay selection [19, 67–69], both the short- and long-lived components are present, and the proper decay-time distribution is a superposition of two exponentials with decay constants $\Gamma_{\rm L,H} = \Gamma_s \pm \Delta\Gamma_s/2$. In principle, this provides sensitivity to both Γ_s and $(\Delta\Gamma_s/\Gamma_s)^2$. Ignoring $\Delta\Gamma_s$ and fitting for a single exponential leads to an estimate of $1/\Gamma_s$ (called effective lifetime) with a relative bias proportional to $(\Delta\Gamma_s/\Gamma_s)^2$. An alternative approach, sensitive to first order in $\Delta\Gamma_s/\Gamma_s$, is to determine the effective lifetime of untagged B^0_s decays to pure CP eigenstates; measurements exist for $B^0_s \to D^+_s D^-_s$ [68], $B^0_s \to K^+K^-$ [69, 70], $B^0_s \to J/\psi\eta$ [71, 72], $B^0_s \to J/\psi f_0(980)$ [73], $B^0_s \to J/\psi\pi^+\pi^-$ [53, 74, 75], $B^0_s \to J/\psi K^0_{\rm S}$ [76], and $B^0_s \to \mu^+\mu^-$ [77]. The extraction of $1/\Gamma_s$ and $\Delta\Gamma_s$ from such measurements, discussed in detail in Ref. [78], requires additional information in the form of theoretical assumptions or external inputs on weak phases and hadronic parameters. In what follows, we only use the effective lifetimes of decays to CP-even ($D^+_s D^-_s$, $J/\psi\eta$) and CP-odd ($J/\psi f_0(980)$, $J/\psi\pi^+\pi^-$) final states where CP conservation can be assumed. In addition $\Delta\Gamma_s$ can be extracted from the decay-time distributions of B^0_s decays to CP-even and CP-odd final states, as has been done by LHCb with $B^0_s \to J/\psi\eta'$ and $B^0_s \to J/\psi\pi^+\pi^-$, respectively [79].

The best sensitivity to $1/\Gamma_s$ and $\Delta\Gamma_s$ is achieved by the time-dependent measurements of the $B^0_s \to J/\psi K^+K^-$ (including $B^0_s \to J/\psi\phi$) and $B^0_s \to \psi(2S)\phi$ decay rates performed at CDF [80], DØ [81], ATLAS [30, 82], CMS [31, 83] and LHCb [27, 60, 84–86], where the CP-even and CP-odd amplitudes are separated statistically through a full angular analysis (see Fig. 75.3). The LHCb collaboration analyzes the $B^0_s \to J/\psi K^+K^-$ decay considering that the K^+K^- system can be in a P-wave or S-wave state, and measures the dependence of the strong phase difference between the P-wave and S-wave amplitudes as a function of the K^+K^- invariant mass [27, 60, 87]; this allows the unambiguous determination of the sign of $\Delta\Gamma_s$, which is found to be positive. All these studies use both untagged and tagged B^0_s candidates and are optimized for the measurement of the phase $\phi_s^{c\bar{c}s}$ that describes CP violation in the interference between B^0_s–$\overline{B}^0_s$ mixing and decay in $b \to c\bar{c}s$ transitions. The published $B^0_s \to J/\psi K^+K^-$, $J/\psi\phi$ and $\psi(2S)\phi$ analyses [27, 30, 31, 60, 80–86] are combined in a multi-dimensional fit including all measured parameters and their correlations. To account for a tension in the time and angular parameters, scale factors are applied on the combined uncertainty of each parameter where a discrepancy arise. For example, the scale factors on the uncertainties of $\Delta\Gamma_s$, Γ_s and $\phi_s^{c\bar{c}s}$ are 1.84, 2.45 and 1.00, respectively. The averages are then further refined by applying constraints from the published lifetime measurements with flavor-specific [19, 67–69] and pure CP [53, 68, 71–75] final states, to yield [8, 88]

$$\Delta\Gamma_s = +0.083 \pm 0.005 \text{ ps}^{-1} \text{ and } 1/\Gamma_s = 1.520 \pm 0.005 \text{ ps}\,, \tag{75.20}$$

or, equivalently,

$$1/\Gamma_{\rm L} = 1.429 \pm 0.006 \text{ ps} \quad \text{and} \quad 1/\Gamma_{\rm H} = 1.622 \pm 0.008 \text{ ps}\,, \tag{75.21}$$

in good agreement with the Standard Model predictions $\Delta\Gamma_s^{\rm SM} = +0.091 \pm 0.015$ ps^{-1} [7] and $\Delta\Gamma_s^{\rm SM} = +0.076 \pm 0.017$ ps^{-1} [89]. Estimates of $\Delta\Gamma_s/\Gamma_s$ obtained from measurements of the $B^0_s \to D_s^{(*)+} D_s^{(*)-}$ branching fractions are not included in the average, since they are based on the questionable [90] assumption that these decays account for all CP-even final states.

From Eqs. (75.5), (75.17) and (75.20), one gets

$$\chi_s = 0.499318 \pm 0.000005\,. \tag{75.22}$$

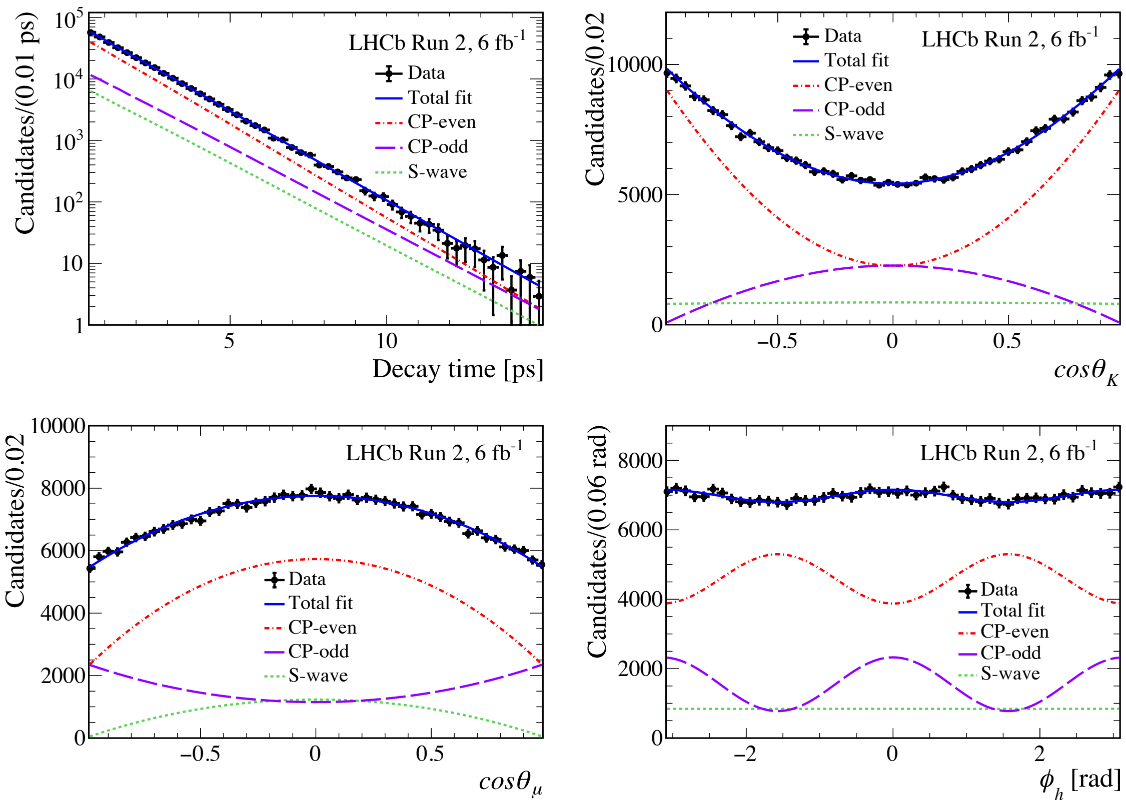

Figure 75.3: Distributions of the proper decay-time and of the three decay angles of background-subtracted $B_s^0 \to J/\psi K^+K^-$ decays reconstructed by LHCb in the vicinity of the ϕ resonance [60]. The kaon pair is predominantly in a P-wave state (from the ϕ decay) which can be CP-even or CP-odd, while a small S-wave state is CP-odd. The curves show the projections of a four-dimensional fit allowing the various components to be disentangled. The different lifetimes of the CP-even and CP-odd components is evident from the decay-time distribution.

75.5 *CP*-violation studies

Evidence for CP violation in B_q^0–$\overline{B}_q^0$ mixing has been searched for, both with flavor-specific and inclusive B_q^0 decays, in samples where the initial flavor state is tagged, usually with a lepton from the other b-hadron in the event. In the case of semileptonic (or other flavor-specific) decays, where the final-state tag is also available, the following asymmetry [2]

$$\mathcal{A}_{\rm SL}^q = \frac{N(\overline{B}_q^0(t) \to \ell^+\nu_\ell X) - N(B_q^0(t) \to \ell^-\overline{\nu}_\ell X)}{N(\overline{B}_q^0(t) \to \ell^+\nu_\ell X) + N(B_q^0(t) \to \ell^-\overline{\nu}_\ell X)} \simeq 1 - |q/p|_q^2 \tag{75.23}$$

has been measured either in time-integrated analyses at CLEO [48, 91], BaBar [92], CDF [93], DØ [94–96] and LHCb [97], or in time-dependent analyses at LEP [38, 98], BaBar [49, 99] and Belle [100]. In the inclusive case, also investigated at LEP [98, 101], no final-state tag is used, and the asymmetry [102]

$$\begin{aligned} &\frac{N(\overline{B}_q^0(t) \to \text{all}) - N(B_q^0(t) \to \text{all})}{N(\overline{B}_q^0(t) \to \text{all}) + N(B_q^0(t) \to \text{all})} \\ &\simeq \mathcal{A}_{\rm SL}^q \left[\sin^2\left(\frac{\Delta m_q\, t}{2}\right) - \frac{x_q}{2}\sin(\Delta m_q\, t)\right] \end{aligned} \tag{75.24}$$

must be measured as a function of the proper time to extract information on CP violation. In addition LHCb has studied the time dependence of the charge asymmetry of $B^0 \to D^{(*)-}\mu^+\nu_\mu X$ decays without tagging the initial state [103], which would be equal to

$$\frac{N(D^{(*)-}\mu^+\nu_\mu X) - N(D^{(*)+}\mu^-\bar{\nu}_\mu X)}{N(D^{(*)-}\mu^+\nu_\mu X) + N(D^{(*)+}\mu^-\bar{\nu}_\mu X)} = \mathcal{A}^d_{\rm SL}\frac{1-\cos(\Delta m_d\, t)}{2} \qquad (75.25)$$

in absence of detection and production asymmetries.

The DØ collaboration measured a like-sign dimuon charge asymmetry in semileptonic b decays that deviates by $2.8\,\sigma$ from the tiny Standard Model prediction and concluded, from a more refined analysis in bins of muon impact parameters, that the overall discrepancy is at the level of $3.6\,\sigma$ [94]. In all other cases, asymmetries compatible with zero (and the Standard Model [7]) have been found, with a precision limited by the available statistics. Several of the analyses at high energy don't disentangle the B^0_d and B^0_s contributions, and either quote a mean asymmetry or a measurement of $\mathcal{A}^d_{\rm SL}$ assuming $\mathcal{A}^s_{\rm SL} = 0$: we no longer include these in the average. An exception is the dimuon DØ analysis [94], which separates the two contributions by exploiting their dependence on the muon impact parameter cut. The resulting measurements of $\mathcal{A}^d_{\rm SL}$ and $\mathcal{A}^s_{\rm SL}$ are then both compatible with the Standard Model. They are also correlated. We therefore perform a two-dimensional average of the measurements of Refs. [48, 49, 91, 92, 94–97, 99, 100, 103] and obtain [8]

$$\mathcal{A}^d_{\rm SL} = -0.0021 \pm 0.0017 \Leftrightarrow |q/p|_d = 1.0010 \pm 0.0008\,, \qquad (75.26)$$

$$\mathcal{A}^s_{\rm SL} = -0.0006 \pm 0.0028 \Leftrightarrow |q/p|_s = 1.0003 \pm 0.0014\,, \qquad (75.27)$$

with a correlation coefficient of -0.054 between $\mathcal{A}^d_{\rm SL}$ and $\mathcal{A}^s_{\rm SL}$. These results show no evidence of CP violation and are compatible with the very small Standard Model predictions, $\mathcal{A}^{d,\rm SM}_{\rm SL} = -(5.1 \pm 0.5) \times 10^{-4}$ and $\mathcal{A}^{s,\rm SM}_{\rm SL} = +(2.2 \pm 0.2) \times 10^{-5}$ [7], but have insufficient precision yet to constrain the Standard Model.

CP violation induced by B^0_s–$\overline{B}^0_s$ mixing in $b \to c\bar{c}s$ decays is controlled by the small weak phase $\phi^{c\bar{c}s}_s$. Measuring $\phi^{c\bar{c}s}_s$ requires tagging the initial flavour of the decaying B^0_s meson. In addition to the previously mentioned $B^0_s \to J/\psi K^+K^-$ (including $B^0_s \to J/\psi\phi$) and $B^0_s \to \psi(2S)\phi$ studies, the decay modes $B^0_s \to J/\psi\pi^+\pi^-$ (including $B^0_s \to J/\psi f_0(980)$) [75, 105] and $B^0_s \to D^+_s D^-_s$ [104] have also been analyzed by LHCb to measure $\phi^{c\bar{c}s}_s$, without the need for an angular analysis. The $J/\psi\pi^+\pi^-$ final state has been shown indeed to be (very close to) a pure CP-odd state [106]. In the $B^0_s \to J/\psi\phi$ and $B^0_s \to J/\psi K^+K^-$ analyses, $\phi^{c\bar{c}s}_s$ is obtained together with several other observables, including $\Delta\Gamma_s$, Γ_s, the longitudinal and perpendicular ϕ polarisation amplitudes, the S-wave amplitude, and strong phases. In order to account for all correlations, the full sets of measurements provided by the different analyses are combined in a multi-dimensional fit [8] of which $\phi^{c\bar{c}s}_s$ is just one of the free parameters. As already mentioned the $B^0_s \to J/\psi\phi$ analyses of ATLAS, CMS and LHCb show a poor overall compatibility, mostly in the lifetime and angular parameters, corresponding approximately to 3 standard deviations. Therefore, scale factors are applied on the combined uncertainty of each parameter where a discrepancy arises. For the parameters already in agreement, such as $\phi^{c\bar{c}s}_s$, no scale factor is applied. The combined result based on all published analyses [27, 30, 31, 60, 75, 80–86, 104, 105] is

$$\phi^{c\bar{c}s}_s = -0.040 \pm 0.016\,. \qquad (75.28)$$

A two-dimensional projection of the overall situation in the $(\phi^{c\bar{c}s}_s, \Delta\Gamma_s)$ plane is shown in Fig. 75.4. The experimental determination of $\phi^{c\bar{c}s}_s$ is still statistically limited. It is consistent with the Standard Model prediction, which is equal to $-2\beta_s = -2\arg(-(V_{ts}V^*_{tb})/(V_{cs}V^*_{cb})) = -0.0367 \pm 0.0010$ [63] or $-0.0376\,^{+0.0006}_{-0.0005}$ [64] assuming negligible Penguin pollution.

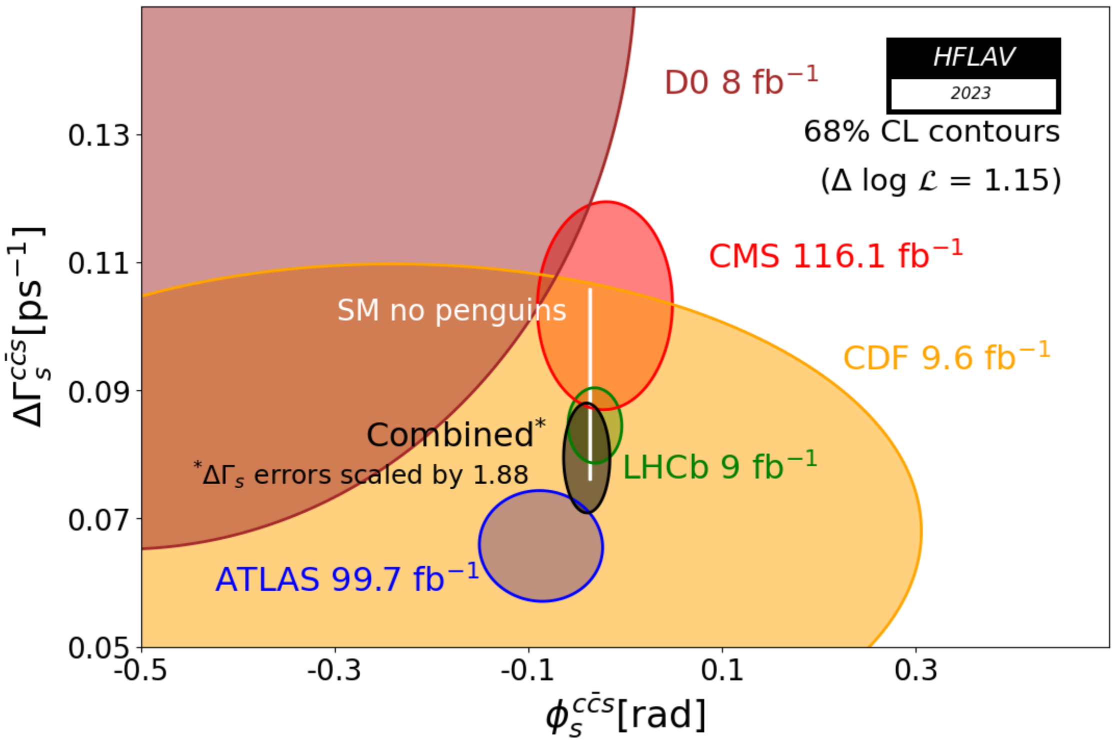

Figure 75.4: 68% CL contours in the $(\phi_s^{c\bar{c}s}, \Delta\Gamma_s)$ plane, showing all measurements from CDF [80], DØ [81], ATLAS [30,82], CMS [31,83] and LHCb [27,60,75,84–86,104,105] using B_s^0 decays governed by the $b \to c\bar{c}s$ transition. Their average [8] is represented as the black ellipse, where the combined uncertainty on $\Delta\Gamma_s$ has been multiplied by 1.88. The very thin white rectangle represents the Standard Model predictions of $-2\beta_s$ [63,64] and $\Delta\Gamma_s$ [7].

CP violation induced by B_s^0–$\overline{B}_s^0$ mixing in $b \to s\bar{s}s$ decays is controlled by the weak phase $\phi_s^{s\bar{s}s}$, which is an observable different from $\phi_s^{c\bar{c}s}$. In the Standard Model, such pure gluonic Penguin decays have an amplitude with a CKM phase that cancels that of the mixing amplitude, and hence $\phi_s^{s\bar{s}s}$ is expected to be essentially zero. LHCb has performed a flavour-tagged time-dependent angular analysis of $B_s^0 \to \phi\phi$ decays [28, 107], similar to that of $B_s^0 \to J/\psi\phi$ decays, and measured

$$\phi_s^{s\bar{s}s} = -0.074 \pm 0.069\,, \tag{75.29}$$

in agreement with the Standard Model prediction.

75.6 Summary

B^0–$\overline{B}^0$ mixing has been and still is a field of intense study. The mass differences in the B_d^0–$\overline{B}_d^0$ and B_s^0–$\overline{B}_s^0$ systems are known to relative precisions of 0.38% and 0.03%, respectively. The non-zero decay width difference in the B_s^0–$\overline{B}_s^0$ system is well established, with a relative difference of $\Delta\Gamma_s/\Gamma_s = (12.7 \pm 0.7)\%$, meaning that the heavy state of the B_s^0–$\overline{B}_s^0$ system lives $\sim$ 14% longer than the light state. In contrast, the relative decay width difference in the B_d^0–$\overline{B}_d^0$ system, $\Delta\Gamma_d/\Gamma_d = (0.1 \pm 1.0)\%$, is still consistent with zero. CP violation in B_d^0–$\overline{B}_d^0$ or B_s^0–$\overline{B}_s^0$ mixing has not been observed yet, with precisions on the semileptonic asymmetries below 0.3%. CP violation induced by B_s^0–$\overline{B}_s^0$ mixing in B_s^0 decays has not yet been observed either, with uncertainties on the $\phi_s^{c\bar{c}s}$ and $\phi_s^{s\bar{s}s}$ phases of 16 mrad and 69 mrad, respectively. All observations so far remain consistent with the Standard Model expectations.

However, the measurements where New Physics might show up are still statistically limited. More results are awaited from the LHC experiments and Belle II, with promising prospects for the investigation of the CP-violating phase $\arg(-M_{12}/\Gamma_{12})$ and improved determination of the $\phi_s^{c\bar{c}s}$ and $\phi_s^{s\bar{s}s}$ phases.

Mixing studies have clearly reached the stage of precision measurements, where much effort is needed, both on the experimental and theoretical sides, in particular to further reduce the hadronic uncertainties of lattice QCD calculations. In the long term, a stringent check of the consistency of the B_d^0 and B_s^0 mixing amplitudes (magnitudes and phases) with all other measured flavor-physics observables will be possible within the Standard Model, leading to very tight limits on (or otherwise a long-awaited surprise about) New Physics.

References

[1] T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Part. Sci. **16**, 511 (1966); I. I. Bigi and A. I. Sanda, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **9**, 1 (2009), second edition, first published in 2000; G. C. Branco, L. Lavoura and J. P. Silva, Int. Ser. Monogr. Phys. **103**, 1 (1999).

[2] See the review on CP violation in the quark sector by T. Gershon and Y. Nir in this publication.

[3] A. J. Buras, W. Slominski and H. Steger, Nucl. Phys. **B245**, 369 (1984).

[4] T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981), erratum *ibid.* **65**, 1772 (1981); for the power-like approximation, see A. J. Buras and R. Fleischer, page 91 in "Heavy Flavours II," eds. A. J. Buras and M. Lindner, Singapore World Scientific (1998).

[5] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[6] I. I. Bigi *et al.*, in "CP violation," ed. C. Jarlskog, Singapore World Scientific (1989).

[7] J. Albrecht, F. Bernlochner, A. Lenz and A. Rusov (2024), [arXiv:2402.04224].

[8] Y. S. Amhis *et al.* (HFLAV), Phys. Rev. D **107**, 052008 (2023), [arXiv:2206.07501]; the combined results on b-hadron lifetimes and mixing parameters published in this *Review* have been obtained by the corresponding sub-group of the Heavy Flavor Averaging (HFLAV) group in March 2024 as updates of the averages published in Chapter 5 of the above paper; for more information, see https://hflav.web.cern.ch/.

[9] C. Albajar *et al.* (UA1), Phys. Lett. **B186**, 247 (1987), erratum *ibid.* **B197**, 565 (1987).

[10] H. Albrecht *et al.* (ARGUS), Phys. Lett. **B192**, 245 (1987).

[11] H. G. Moser and A. Roussarie, Nucl. Instrum. Meth. **A384**, 491 (1997).

[12] SLD collab., SLAC-PUB-7228, SLAC-PUB-7229, and SLAC-PUB-7230, *28th Int. Conf. on High Energy Physics,* Warsaw (1996); J. L. Wittlin, *A measurement of the time dependence of B_d^0–$\overline{B}_d^0$ mixing with kaon tagging*, Ph.D. thesis, Massachusetts U., Amherst (2001), URL `http://wwwlib.umi.com/dissertations/fullcit?p3027272`.

[13] ALEPH collab., contrib. 596 to *Int. Europhysics Conf. on High Energy Physics,* Jerusalem (1997).

[14] K. Abe *et al.* (SLD), Phys. Rev. **D67**, 012006 (2003), [hep-ex/0209002].

[15] J. Abdallah *et al.* (DELPHI), Eur. Phys. J. **C28**, 155 (2003), [hep-ex/0303032].

[16] F. Abe *et al.* (CDF), Phys. Rev. Lett. **80**, 2057 (1998), [hep-ex/9712004]; F. Abe *et al.* (CDF), Phys. Rev. **D59**, 032001 (1999), [hep-ex/9806026]; F. Abe *et al.* (CDF), Phys. Rev. **D60**, 051101 (1999); F. Abe *et al.* (CDF), Phys. Rev. **D60**, 072003 (1999), [hep-ex/9903011]; T. Affolder *et al.* (CDF), Phys. Rev. **D60**, 112004 (1999), [hep-ex/9907053].

[17] R. Barate *et al.* (ALEPH), Eur. Phys. J. **C4**, 367 (1998); R. Barate *et al.* (ALEPH), Eur. Phys. J. **C7**, 553 (1999), [hep-ex/9811018].

[18] R. Aaij *et al.* (LHCb), JINST **10**, P10005 (2015), [arXiv:1507.07892].

[19] P. Abreu *et al.* (DELPHI), Eur. Phys. J. **C16**, 555 (2000), [hep-ex/0107077].

[20] K. Anikeev *et al.* (2002), [hep-ph/0201071].

[21] A. Abulencia *et al.* (CDF), Phys. Rev. Lett. **97**, 242003 (2006), [hep-ex/0609040].

[22] V. M. Abazov *et al.* (DØ), Phys. Rev. **D74**, 112002 (2006), [hep-ex/0609034].

[23] V. M. Abazov *et al.* (DØ), Phys. Rev. Lett. **101**, 241801 (2008), [arXiv:0802.2255].

[24] R. Aaij *et al.* (LHCb), Eur. Phys. J. **C72**, 2022 (2012), [arXiv:1202.4979].

[25] R. Aaij *et al.* (LHCb), JINST **11**, P05010 (2016), [arXiv:1602.07252].

[26] R. Aaij *et al.* (LHCb), Eur. Phys. J. **C77**, 238 (2017), [arXiv:1610.06019].

[27] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **114**, 041801 (2015), [arXiv:1411.3104].

[28] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **131**, 17, 171802 (2023), [arXiv:2304.06198].

[29] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **117**, 261801 (2016), [arXiv:1608.06620].

[30] G. Aad *et al.* (ATLAS), Eur. Phys. J. **C81**, 342 (2021), [arXiv:2001.07115].

[31] A. M. Sirunyan *et al.* (CMS), Phys. Lett. **B816**, 136188 (2021), [arXiv:2007.02434].

[32] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **94**, 161803 (2005), [hep-ex/0408127]; K. F. Chen *et al.* (Belle), Phys. Rev. **D72**, 012004 (2005), [hep-ex/0504023].

[33] Throughout this document we omit references of results that have been replaced by new published measurements.

[34] D. Buskulic *et al.* (ALEPH), Z. Phys. **C75**, 397 (1997).

[35] P. Abreu *et al.* (DELPHI), Z. Phys. **C76**, 579 (1997).

[36] M. Acciarri *et al.* (L3), Eur. Phys. J. **C5**, 195 (1998).

[37] G. Alexander *et al.* (OPAL), Z. Phys. **C72**, 377 (1996); K. Ackerstaff *et al.* (OPAL), Z. Phys. **C76**, 417 (1997), [hep-ex/9707010]; G. Abbiendi *et al.* (OPAL), Phys. Lett. **B493**, 266 (2000), [hep-ex/0010013].

[38] K. Ackerstaff *et al.* (OPAL), Z. Phys. **C76**, 401 (1997), [hep-ex/9707009].

[39] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **88**, 221802 (2002), [hep-ex/0112044]; B. Aubert *et al.* (BaBar), Phys. Rev. **D66**, 032003 (2002), [hep-ex/0201020]; B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **88**, 221803 (2002), [hep-ex/0112045]; B. Aubert *et al.* (BaBar), Phys. Rev. **D67**, 072002 (2003), [hep-ex/0212017]; B. Aubert *et al.* (BaBar), Phys. Rev. **D73**, 012004 (2006), [hep-ex/0507054].

[40] N. C. Hastings *et al.* (Belle), Phys. Rev. **D67**, 052004 (2003), [hep-ex/0212033]; Y. Zheng *et al.* (Belle), Phys. Rev. **D67**, 092004 (2003), [hep-ex/0211065]; K. Abe *et al.* (Belle), Phys. Rev. **D71**, 072003 (2005), erratum *ibid.* **D71**, 079903 (2005), [hep-ex/0408111].

[41] F. Abudinén *et al.* (Belle II), Phys. Rev. **D107**, L091102 (2023), [arXiv:2302.12791].

[42] R. Aaij *et al.* (LHCb), Phys. Lett. **B709**, 177 (2012), [arXiv:1112.4311].

[43] R. Aaij *et al.* (LHCb), Phys. Lett. **B719**, 318 (2013), [arXiv:1210.6750].

[44] R. Aaij *et al.* (LHCb), Eur. Phys. J. **C73**, 2655 (2013), [arXiv:1308.1302].

[45] R. Aaij *et al.* (LHCb), Eur. Phys. J. **C76**, 412 (2016), [arXiv:1604.03475].

[46] H. Albrecht *et al.* (ARGUS), Z. Phys. **C55**, 357 (1992); H. Albrecht *et al.* (ARGUS), Phys. Lett. **B324**, 249 (1994).

[47] J. E. Bartelt *et al.* (CLEO), Phys. Rev. Lett. **71**, 1680 (1993).

[48] B. H. Behrens *et al.* (CLEO), Phys. Lett. **B490**, 36 (2000), [hep-ex/0005013].

[49] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **92**, 181801 (2004), [hep-ex/0311037]; B. Aubert *et al.* (BaBar), Phys. Rev. **D70**, 012007 (2004), [hep-ex/0403002].

[50] T. Higuchi *et al.*, Phys. Rev. **D85**, 071105 (2012), [arXiv:1203.0930].

[51] R. Aaij *et al.* (LHCb), JHEP **04**, 114 (2014), [arXiv:1402.2554].

[52] M. Aaboud *et al.* (ATLAS), JHEP **06**, 081 (2016), [arXiv:1605.07485].

[53] A. M. Sirunyan *et al.* (CMS), Eur. Phys. J. **C78**, 457 (2018), erratum *ibid.* **C78**, 561 (2018), [arXiv:1710.08949].

[54] M. Artuso, G. Borissov and A. Lenz, Rev. Mod. Phys. **88**, 045002 (2016), [arXiv:1511.09466].

[55] See the review on the CKM quark-mixing matrix by A. Ceccucci, Z. Ligeti, and Y. Sakai in this publication.

[56] S. Aoki *et al.* (Flavour Lattice Averaging Group) (2019), [arXiv:1902.08191].

[57] R. Aaij *et al.* (LHCb), New J. Phys. **15**, 053021 (2013), [arXiv:1304.4741].

[58] R. Aaij *et al.* (LHCb), JHEP **03**, 137 (2021), [arXiv:2011.12041].

[59] R. Aaij *et al.* (LHCb), Nature Phys. **18**, 1 (2022), [arXiv:2104.04421].

[60] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **132**, 051802 (2024), [arXiv:2308.01468].

[61] A. Bazavov *et al.* (Fermilab Lattice and MILC), Phys. Rev. **D93**, 113016 (2016), [arXiv:1602.03560].

[62] D. King, A. Lenz and T. Rauh, JHEP **05**, 034 (2019), [arXiv:1904.00940].

[63] M. Bona *et al.* (UTfit), Rend. Lincei Sci. Fis. Nat. **34**, 37 (2023), [arXiv:2212.03894]; updated results at http://www.utfit.org/.

[64] J. Charles *et al.* (CKMfitter), Phys. Rev. **D91**, 073007 (2015), [arXiv:1501.05013]; updated results at http://ckmfitter.in2p3.fr/.

[65] K. Hartkorn and H. G. Moser, Eur. Phys. J. **C8**, 381 (1999).

[66] M. Acciarri *et al.* (L3), Phys. Lett. **B438**, 417 (1998).

[67] D. Buskulic *et al.* (ALEPH), Phys. Lett. **B377**, 205 (1996); K. Ackerstaff *et al.* (OPAL), Phys. Lett. **B426**, 161 (1998), [hep-ex/9802002]; F. Abe *et al.* (CDF), Phys. Rev. **D59**, 032004 (1999), [hep-ex/9808003]; V. M. Abazov *et al.* (DØ), Phys. Rev. Lett. **114**, 062001 (2015), [arXiv:1410.1568]; T. Aaltonen *et al.* (CDF), Phys. Rev. Lett. **107**, 272001 (2011), [arXiv:1103.1864]; R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **113**, 172001 (2014), [arXiv:1407.5873]; R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **119**, 101801 (2017), [arXiv:1705.03475].

[68] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **112**, 111802 (2014), [arXiv:1312.1217].

[69] R. Aaij *et al.* (LHCb), Phys. Lett. **B736**, 446 (2014), [arXiv:1406.7204].

[70] R. Aaij *et al.* (LHCb), Phys. Lett. **B707**, 349 (2012), [arXiv:1111.0521].

[71] R. Aaij *et al.* (LHCb), Phys. Lett. **B762**, 484 (2016), [arXiv:1607.06314].

[72] R. Aaij *et al.* (LHCb), Eur. Phys. J. **C83**, 7, 629 (2023), [arXiv:2206.03088].

[73] T. Aaltonen *et al.* (CDF), Phys. Rev. **D84**, 052012 (2011), [arXiv:1106.3682]; V. M. Abazov *et al.* (DØ), Phys. Rev. **D94**, 012001 (2016), [arXiv:1603.01302].

[74] R. Aaij *et al.* (LHCb), Phys. Rev. **D87**, 112010 (2013), [arXiv:1304.2600].

[75] R. Aaij *et al.* (LHCb), Phys. Lett. **B797**, 134789 (2019), [arXiv:1903.05530].

[76] R. Aaij *et al.* (LHCb), Nucl. Phys. **B873**, 275 (2013), [arXiv:1304.4500].

[77] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **128**, 041801 (2022), [arXiv:2108.09284]; R. Aaij *et al.* (LHCb), Phys. Rev. **D105**, 012010 (2022), [arXiv:2108.09283]; A. Tumasyan *et al.* (CMS), Phys. Lett. **B842**, 137955 (2023), [arXiv:2212.10311]; G. Aad *et al.* (ATLAS), JHEP **09**, 199 (2023), [arXiv:2308.01171].

[78] R. Fleischer and R. Knegjens, Eur. Phys. J. **C71**, 1789 (2011), [arXiv:1109.5115].

[79] R. Aaij *et al.* (LHCb), submitted to JHEP, [arXiv:2310.12649].

[80] T. Aaltonen *et al.* (CDF), Phys. Rev. Lett. **109**, 171802 (2012), [arXiv:1208.2967].

[81] V. M. Abazov *et al.* (DØ), Phys. Rev. **D85**, 032006 (2012), [arXiv:1109.3166].

[82] G. Aad *et al.* (ATLAS), Phys. Rev. **D90**, 052007 (2014), [arXiv:1407.1796]; G. Aad *et al.* (ATLAS), JHEP **08**, 147 (2016), [arXiv:1601.03297].

[83] V. Khachatryan *et al.* (CMS), Phys. Lett. **B757**, 97 (2016), [arXiv:1507.07527].

[84] R. Aaij *et al.* (LHCb), Eur. Phys. J. **C81**, 1026 (2021), [arXiv:2105.14738].

[85] R. Aaij *et al.* (LHCb), JHEP **08**, 037 (2017), [arXiv:1704.08217].

[86] R. Aaij *et al.* (LHCb), Phys. Lett. **B762**, 253 (2016), [arXiv:1608.04855].

[87] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **108**, 241801 (2012), [arXiv:1202.4717].

[88] Including the result of Ref. [79], which has not been accepted by a journal at the time of writing this review, changes $\Delta\Gamma_s$ to $+0.084 \pm 0.004$ ps^{-1} and $1/\Gamma_{\rm H}$ to 1.623 ± 0.008 ps.

[89] M. Gerlach, U. Nierste, P. Reeck, V. Shtabovenko and M. Steinhauser (2024), [arXiv:2403.08316].

[90] A. Lenz and U. Nierste, JHEP **06**, 072 (2007), [hep-ph/0612167].

[91] D. E. Jaffe *et al.* (CLEO), Phys. Rev. Lett. **86**, 5000 (2001), [hep-ex/0101006].

[92] J. P. Lees *et al.* (BaBar), Phys. Rev. Lett. **114**, 081801 (2015), [arXiv:1411.1842].

[93] F. Abe *et al.* (CDF), Phys. Rev. **D55**, 2546 (1997).

[94] V. M. Abazov *et al.* (DØ), Phys. Rev. **D89**, 012002 (2014), [arXiv:1310.0447].

[95] V. M. Abazov *et al.* (DØ), Phys. Rev. **D86**, 072009 (2012), [arXiv:1208.5813].

[96] V. M. Abazov *et al.* (DØ), Phys. Rev. Lett. **110**, 011801 (2013), [arXiv:1207.1769].

[97] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **117**, 061803 (2016), erratum *ibid.* **118**, 129903 (2017), [arXiv:1605.09768].

[98] R. Barate *et al.* (ALEPH), Eur. Phys. J. **C20**, 431 (2001).

[99] J. P. Lees *et al.* (BaBar), Phys. Rev. Lett. **111**, 101802 (2013), erratum *ibid.* **111**, 159901 (2013), [arXiv:1305.1575].

[100] E. Nakano *et al.* (Belle), Phys. Rev. **D73**, 112002 (2006), [hep-ex/0505017].

[101] G. Abbiendi *et al.* (OPAL), Eur. Phys. J. **C12**, 609 (2000), [hep-ex/9901017].

[102] M. Beneke, G. Buchalla and I. Dunietz, Phys. Lett. **B393**, 132 (1997), [hep-ph/9609357]; I. Dunietz, Eur. Phys. J. **C7**, 197 (1999), [hep-ph/9806521].

[103] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **114**, 041601 (2015), [arXiv:1409.8586].

[104] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **113**, 211801 (2014), [arXiv:1409.4619].

[105] R. Aaij *et al.* (LHCb), Phys. Lett. **B736**, 186 (2014), [arXiv:1405.4140].

[106] R. Aaij *et al.* (LHCb), Phys. Rev. **D86**, 052006 (2012), [arXiv:1204.5643].

[107] R. Aaij *et al.* (LHCb), Phys. Rev. **D90**, 052011 (2014), [arXiv:1407.2222].